SUMMARY  This paper presents the bit error rate (BER) upper bounds for trellis coded asymmetric 8PSK (TC-A8PSK) system using the Ka-band satellite in the rain fading environment. The probability density function (PDF) for the rain fading random variable can be theoretically derived by assuming that the rain attenuation can be approximated to a log-normal distribution and the rain fading parameters are calculated by using the rain precipitation data from the Crane global model. Furthermore, we analyze the BER upper bounds of TC-A8PSK system according to the number of states in the trellis diagram and the availability of channel state information (CSI). In the past, Divsalar and Simon [9] has analyzed the BER upper bounds of 2-state TCM system in Rician fading channels; however, this paper is the first to analyze the BER upper bounds of TCM system in the rain fading channels. Finally, we summarize the dominant six factors which are closely related to the BER upper bounds of TC-A8PSK satellite system in the rain fading channel as follows: (1) frequency band, (2) rain intensity, (3) elevation angle, (4) bit energy to noise ratio, (5) asymmetric angle, and (6) availability of CSI.

key words: trellis coded asymmetric 8PSK (TC-A8PSK), rain fading, Ka-band satellite communication

1. Introduction

The unique features of satellite communications are their ability to overcome the geographical limitations and to provide the new service at wide areas. Recently, the high frequency bands including Ka-band are being highly developed because of the increasing demand of broadband services and the shortage of conventional frequency bands. However, Ka-band satellite communication systems using the 20/30 GHz are subject to severe performance degradations due to the atmospheric propagation fading. The principal causes of this unwanted signal attenuation are the variations in water vapor along a slant path or the precipitation of rain. Specifically, the rain attenuation is so serious as to cause link outage.

For mobile satellite communication systems, multipath fading produces a received signal with amplitude fluctuation which can be modeled as a Rician random variable. If shadowing is severe, then a Rayleigh statistical model becomes appropriate [1]. However, the rain attenuation due to rain fading is nearly proportional to the $n$-th power of rain intensity [2]. Although the prediction of rain attenuation is a statistical process, many models have been developed which yield results that agree well with experimental observations. The major models are those of Rice-Holmberg [3], Lin [4], Crane [5], and Morita [6] etc. In particular, Morita model is based on the fact that when rain intensity is well approximated by log-normal distribution, then rain attenuation can be also approximated by log-normal distribution.

As indicated in [7], the constraint on the downlink flux density and the requirement of a small receiving antenna aperture make a mobile satellite channel power limited. On the other hand, to serve a large number of users in a given bandwidth, it is inevitably band limited. For this reason, trellis coded modulation (TCM) is an appropriate coding and modulation scheme for these channels since it is both power and bandwidth efficient. TCM schemes were first proposed in [8] and allow the achievement of good coding gain by treating coding and modulation as a single entity. In other words, if a well chosen $n/(n + 1)$ code can be combined with a bandwidth efficient $2^{n+1}$-ary modulation scheme, the bandwidth expansion from coding can be negated by the bandwidth reduction from the $2^{n+1}$-ary modulation. In addition, it is known that symmetric signal sets, which are optimum in uncoded additive white Gaussian noise (AWGN) environments, are not necessarily optimum for coded systems [9]. For example, by designing asymmetric 8PSK signal constellations and combining them with 2/3 trellis coding, one can further improve the performance of coded systems without increasing power or changing the bandwidth constraints imposed on the system. In this paper, we also extend this concept to trellis coded asymmetric 8PSK (TC-A8PSK) systems in the rain fading channels (see Figs. 10, 12, and 14).

This paper presents the bit error rate (BER) upper bounds of TC-A8PSK system using the Ka-band satellite in the rain fading channels. The probability density function (PDF) for rain fading random variable can be theoretically derived by using log-normal rain intensity distribution. And the rain fading parameters are calculated by using the rain precipitation data from the Crane global model. Furthermore, we analyze the BER upper bounds of TC-A8PSK system according to the number of states in the trellis diagram and the availability of channel state information (CSI). Finally, we summarize the dominant factors which are closely re-
lated to the BER upper bounds of TC-8PSK system in the rain fading channel.

The organization of this paper is as follows. In the next section, we describe an equivalent baseband system model of the TC-A8PSK system and a rain fading channel model. In Sect. 3, we describe the basic parameters and the pairwise error probability. Then we analyze the average branch gain factors in the presence of rain fading. In Sect. 4, we derive the BER upper bounds of 2- and 4-states TC-A8PSK system in the rain fading channel. In Sect. 5, we show numerical results of our analysis according to dominant parameters. We compare these results with the BER upper bounds of 2-state TCM system which was derived by Divsalar and Simon [9] in the Rician fading channels, and in Sect. 6, we draw some conclusions.

2. System and Channel Model

2.1 System Model

A block diagram of trellis coded asymmetric 8PSK (TC-A8PSK) system under investigation is illustrated in Fig. 1. A rate 2/3 convolutional encoder is implemented with a combination of n shift registers (the memory of encoding operation) and mod-2 adders. The data stream enters a rate 2/3 convolutional encoder. The encoder output symbols are then block interleaved and mapped into an asymmetric 8PSK signal set. The transmitted signals are impaired by rain fading distortions and AWGN. The received signals enter asymmetric demodulator. And then through the block deinterleaver and the channel state estimator, the demodulated signals enter Viterbi decoder. Channel state information (CSI) can be derived from the received signal to help the Viterbi decoder in improving its performance. Of course, the metric calculation of the Viterbi decoder depends on whether or not CSI is provided.

Throughout this paper, for the purpose of utilizing of well known analytical approach [9], the infinite interleaving depth will be assumed. This assumption means that the channel under consideration is memoryless. In fact, rain fading is a slow process so that the correlation between the fades cannot be removed by interleaving. For this reason, power control is used to combat the log-normal fading (rain fading, shadowing etc.) and path loss. Therefore, these results will be slightly optimistic when compared to those derived from simulations. Infinite traceback depth in the Viterbi decoding process and perfect coherent detection will also be assumed.

2.2 Channel Model

The two most significant propagation aspects of the mobile satellite radio channel are shadowing and multipath fading. An appropriate model was proposed in [1], which assumes that the line of sight (LOS) component under foliage attenuation (shadowing) is log-normally distributed and that the multipath effect is Rayleigh distributed. However, Ka-band satellite radio channels using the band of 20/30 GHz are more susceptible to the atmospheric propagation fading, which can directly impact service availability.

In this paper, rain attenuation is calculated by using the Morita model [6], which is based on the fact that when rain intensity is well approximated by log-normal distribution, then rain attenuation can be better approximated by log-normal distribution, in particular, for the rain rate less than about 50 mm/h. We derive the probability density function (PDF) for the rain fading random variable from the log-normal rain attenuation distribution. In addition, the derivation of PDF for the rain fading random variable is described in APPENDIX. The PDF $f_{\rho}(\rho)$ for the rain fading amplitude $\rho$ is given by

$$ f_{\rho}(\rho) = \begin{cases} 2C \frac{e^{- \left( \log_{10}(\rho) - m_A \right)^2 / 2\sigma_r^2}}{\sqrt{2\pi s_A \rho}} & \rho > 0 \\ 0; & \text{otherwise} \end{cases} \tag{1} $$

where $C = \log_{10} e \simeq 0.4343$, $e$ being the base of natural logarithm, and $m_A$ and $s_A$ are the average value and the standard deviation of $\log_{10} A$, respectively, where $A$ is the rain attenuation. The parameters of rain attenuation $m_A$, $s_A^2$ can be given by [6]

$$ m_A = \log_{10} \mu - \frac{1}{2} \log_{10} \left( 1 + \frac{\sigma^2}{\mu^2} \right), $$

$$ s_A^2 = C \log_{10} \left( 1 + \frac{\sigma^2}{\mu^2} \right), \tag{2} $$

where $\mu$, $\sigma^2$ are the average value and the variance of rain attenuation $A$, respectively, and can be given by

$$ \mu = D \exp \left( \frac{\log_{10} a + bm_r}{C} + \frac{(bs_r)^2}{2C^2} \right), $$
Table 1  Parameters of rain intensity and rain attenuation of Crane global model.

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<tr>
<td>Rain Intensity ((\mu))</td>
<td>-0.4157</td>
<td>-0.4869</td>
<td>-0.5899</td>
<td>-0.4703</td>
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<tr>
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<td>-0.3405</td>
<td>-0.2353</td>
<td>-0.0942</td>
<td>0.0537</td>
<td>0.0213</td>
<td>-0.8064</td>
</tr>
<tr>
<td>Correlation Coefficient</td>
<td>0.9988</td>
<td>0.9972</td>
<td>0.9946</td>
<td>0.9996</td>
<td>0.9990</td>
<td>0.9980</td>
<td>0.9878</td>
<td>0.9984</td>
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\[
\sigma^2 = \frac{4}{\alpha^2} \left\{ D + 2 \left( D + \frac{3\sqrt{D}}{\alpha} + \frac{3}{\alpha} \right) \right\} \\
\times e^{-\alpha \sqrt{D} - \frac{6}{\alpha^2}} \\
\times \exp \left\{ \frac{2(\log_{10} A + bm_r)}{C} + \left( \frac{bs_r}{C} \right)^2 \right\} \\
\times \left\{ \exp \left( \frac{bs_r}{C} \right)^2 - 1 \right\}.
\]

(3)

In (3), \(\alpha\) is the rain intensity spatial correlation over slant path, \(a\) and \(b\) are the parameters which depend on radio frequency and polarization (in this paper, this parameters reflect the Ka-band frequency), \(\varepsilon\) is elevation angle, \(D\) is the length of the rainy path, \(m_r\) and \(s_r\) are the average value and the standard deviation of \(\log_{10} r\), respectively, where \(r\) is the rain intensity. In the paper, the parameters become \(\alpha = 0.35\), \(a = 0.225\), \(b = 0.88\), \(\varepsilon = 45^\circ\), and 0° C layer height of 4 km for the 21 GHz horizontally polarized wave.

We analyze the BER upper bounds in the rain fading channels by considering the Crane model [5], which provides eight rain climate regions \(A\) through \(H\) covering the whole globe. The parameters of rain intensity \(m_r, s_r\) can be found by linear regression analysis of rain intensity and percent of year pairs. To calculate the parameters of rain intensity, first of all, we should define the percent of year (%). The percent of year represents an estimate of a total time, over a 1-year period, that rain intensity can be expected to exceed a specified amount. For more detailed presentation, readers may refer to [2]. The parameters of the rain intensity \((m_r, s_r)\) and rain attenuation \((m_A, s_A)\) are presented in Table 1 for eight rain climate regions of Crane global model. The parameters of the rain attenuation are calculated at the elevation angle of 45°. In Table 1, the correlation coefficient represents the dependence between rain intensity and percent of year pairs. The value of correlation coefficient lies between \(-1\) and 1, inclusive. As a result, the means of \(\log_{10} A\) for dry climate regions (regions \(A, B, C,\) and \(F\)) are smaller than those for wet climate regions (regions \(D,\) \(E, G,\) and \(H\)). The PDFs of the rain attenuation and the rain fading for rain climate region \(D\) are presented in Fig. 2. The simulated and modeled PDF are also compared in this figure. The PDF of the rain attenuation provides the good match with typical log-normal distribution. The variations of the rain fading channel parameters, \(m_A\) and \(s_A\), are presented in Fig. 3 as a function of elevation angle for rain climate region \(D\). The smaller the elevation angle, the smaller the standard deviation \(s_A\), whereas the larger the mean \(m_A\). Therefore, the rain attenuation can reach to significant levels, in particular, at the elevation angles below 30°. From the above results, we can find the elevation angle dependence of rain fading channel parameters.
transmitted coded sequence of length $9$. In addition, we show the average branch gain factuating the BER upper bounds of a TC-A8PSK system.

3. Derivation of the Pairwise Error Probability Bounds

In this section, we give a brief review of the basic analysis procedure of pairwise error probability used in evaluating the BER upper bounds of a TC-A8PSK system [9]. In addition, we show the average branch gain factors according to the availability of CSI. We denote a transmitted coded sequence of length $N$ by

$$\mathbf{x} = (x_1, x_2, \cdots, x_N),$$

where $x_k$ is the $k$-th complex number representing the 8PSK signals and is a nonlinear function of the state of the encoder and the $n$ information input bits.

At the receiving end, corresponding to the transmitted coded sequence $\mathbf{x}$, the received sequence $\mathbf{y} = (y_1, y_2, \cdots, y_N)$ appears at the output of the channel, where $y_k, k = 1, 2, \cdots, N$ is given by

$$y_k = \rho_k x_k + n_k.$$  

In (5), $\rho_k$ is a real random variable representing the effect of the rain fading channel on the received amplitude and $n_k$ is an additive white Gaussian random variable with zero mean and variance $\sigma_n^2$. Next, note the pairwise error probability $P(\mathbf{x} \rightarrow \mathbf{x}')$, which represents the probability of choosing the coded sequence $\mathbf{x}'$ when indeed $\mathbf{x}$ was transmitted, with $\mathbf{x}'$ and $\mathbf{x}$ being the only choices. Using the symbol metrics, we can show that the pairwise error probability is given by

$$P(\mathbf{x} \rightarrow \mathbf{x}') = \Pr \{ m(\mathbf{y}, \mathbf{x}') \geq m(\mathbf{y}, \mathbf{x}) / \mathbf{x} \},$$

where

$$m(\mathbf{y}, \mathbf{x}) = \sum_{k=1}^{N} m(y_k, x_k).$$

Applying the Chernoff bound, (6) becomes

$$P(\mathbf{x} \rightarrow \mathbf{x}') = \prod_{k \in \eta} E \{ \exp (\lambda [m(y_k, x_k') \geq m(y_k, x_k)] / \mathbf{x} \}},$$

where $E \{ \cdot \}$ is the statistical expectation operation, $\lambda (\lambda \geq 0)$ is the Chernoff parameter, $\eta$ and is the set of all $k$ such that $x_k \neq x_k'$. The symbol metrics at time $k$ according to the availability of channel state information are given by

$$m(y_k, x_k) = \left\{ \begin{array}{ll} |y_k - \rho_k x_k|^2, & \text{ideal CSI} \\ |y_k - x_k|^2, & \text{no CSI} \end{array} \right..$$

Thus, substituting (9) into (8), optimizing (8) over the PDF of rain fading random variable $\rho$ (this averaging will be denoted by $E_{\text{Rain}(\cdot)}$), we can get the unconditional pairwise error probability which is mathematically described by

$$P(\mathbf{x} \rightarrow \mathbf{x}') \leq E_{\text{Rain}}(D^2(x, x')), \quad D = \exp(-1/4\sigma_n^2),$$

where $d^2(x, x')$ represents the square of the weighted Euclidean distances between the two symbol sequences $x'$ and $x$, and can be expressed in the form

$$d^2(x, x') = \sum_{k \in \eta} w_k^2 \delta_k^2,$$

where $\delta_k^2 = |x_k - x_k'|^2$ and $w_k^2$ can be expressed according to the availability of channel state information as follows:

$$w_k^2 = \left\{ \begin{array}{ll} \rho_k^2, & \text{ideal CSI} \\ 4\lambda(\rho_k - \lambda), & \text{no CSI} \end{array} \right..$$

For 8PSK TCM schemes, $D$ of (10) can be rewritten as

$$D = \exp \left( -\frac{E_s}{4N_o} \right) = \exp \left( -\frac{E_b}{2N_o} \right).$$

Next, a typical asymmetric 8PSK signal sets and the associated distances (scalar products) are illustrated in Fig.4. Considering the asymmetric signal sets, the squared distance $\delta_k^2$ from signal point 0 to signal point $k = 1, 2, \cdots, 7$ can be given by

$$\delta_0^2 = 4(1 - \cos \phi), \quad \delta_1^2 = 2,$$
$$\delta_2^2 = 4(1 + \cos \phi), \quad \delta_3^2 = 4,$$
$$\delta_4^2 = 4(1 + \cos \phi), \quad \delta_5^2 = \delta_2^2 = 2,$$
$$\delta_6^2 = 2(1 - \sin \phi),$$
$$\delta_7^2 = 2(1 - \sin \phi).$$
where $\phi$ is an asymmetric angle of TC-A8PSK system. It is known that symmetric signal sets, which are optimum in uncoded AWGN environments, are not necessarily optimum for coded systems [9]. Hence, by using asymmetric signal sets, one can obtain a performance gain over the traditional symmetric constellations combined with trellis coding.

We now consider the average branch gain factors, denoted $E_{\text{Rain}}(D w^2 \delta_i^2)$, which are calculated by substituting (11) into (10). The average branch gain factors according to the availability of CSI in Crane region $D$ are presented in Fig. 5. In this case, we assume that both the elevation angle $\varepsilon$ and the asymmetric angle $\phi$ are $45^\circ$. As expected, the lack of channel state information produces noticeable degradation in average branch gain factors. Furthermore, these results indicate that the larger the bit energy to noise ratio, the smaller the average branch gain factor. Likewise, the larger the squared distance, the smaller the average branch gain factor and vice versa. If the squared distance is equal, the average branch gain factor is also equal. Then, we can show that

\[
E_{\text{Rain}}(D w^2 \delta_i^2) \leq E_{\text{Rain}}(D w^2 \delta_j^2) \leq E_{\text{Rain}}(D w^2 \delta_k^2) 
\leq E_{\text{Rain}}(D w^2 \delta_l^2) \leq E_{\text{Rain}}(D w^2 \delta_m^2) = E_{\text{Rain}}(D w^2 \delta_n^2) = E_{\text{Rain}}(D w^2 \delta_o^2) = E_{\text{Rain}}(D w^2 \delta_p^2) (15)
\]

Consequently, the additional coding gain due to asymmetry can be obtained by selecting the asymmetric angle that maximizes the minimum Euclidean distance.

In the sequel, for the case of no channel state information, we must minimize the upper bound of (10) over the Chernoff parameter $\lambda$, which is a non-negative value, to obtain the tightest BER upper bound. The optimum $\lambda$'s which satisfy this minimization are presented in Fig. 6, where we use the same basic parameters as previously assumed in Fig. 5. As a result, the larger the bit energy to noise ratio, the smaller the optimum Chernoff parameter. Likewise, the larger the squared distance, the smaller the optimum Chernoff parameter and vice versa.
4. Derivation of the Bit Error Probability Bounds

To derive the BER upper bounds from the pairwise error probability bounds in the rain fading channels, we follow the transfer function approach taken in [9]. The average bit error rate $P_b$ in the rain fading channel is upper bounded by

$$P_b \leq \left( \min_{\lambda \geq 0} \left( \min_{0 \leq \phi \leq 90^\circ} \right) \right) \frac{1}{n} \left\| \frac{\partial T(E_{\text{Rain}}(D^{w^2\phi^2}), I)}{\partial I} \right\|_{D=\exp(-E_{\phi}/4N_0}, I=1}$$

where $n$ is the number of information bits of a rate $n/(n+1)$ trellis encoder and $T$ is the transfer function of the error state diagram. The minimization of the BER upper bound with respect to Chernoff parameter $\lambda$ is considered only for the case of no channel state information, and the minimization of the BER upper bound with respect to asymmetric angle $\phi$ is considered only for the case of the number of states in the trellis diagram which produce a coding gain due to asymmetry.

In (16), the transfer function $T(E_{\text{Rain}}(D^{w^2\phi^2}), I)$ of the error state diagram is easily derived by using the properties of isometric mapping [10]. If the mapping $g(c) \rightarrow g(c \oplus c')$, where $c \in C_0$ and $c' \notin C_0$, is one-to-one and preserves the distances, then the mapping is called an isometry. Here the set $C_0$ consists of half of the total $2^{n+1}$ possible output states for the encoder. In this paper, we present the BER upper bounds of 2- and 4-state TC-A8PSK system, respectively, and compare the performance between them.

4.1 2-State TC-A8PSK System

Figure 7(a) shows the 2-state TC-A8PSK encoder. Since we can obtain the gain due to asymmetry in the 2-state TC-A8PSK system, we need to find the optimum asymmetric angle $\phi$ which minimizes (16). The corresponding trellis diagram is presented in Fig. 7(b). Since there exists the parallel transition, we should consider the parallel transition to determine the transfer function of the error state diagram. The error state diagram of the 2-state TC-A8PSK system is illustrated in Fig. 7(c). Using the Mason’s rule [10], we obtain the following transfer function for the graph.

$$T(D, I) = IE_{\text{Rain}}(D^{w^2\phi^2}) + \frac{ac}{1-b},$$

where the graph labels $a$, $b$, and $c$ of (17) are calculated by using the properties of isometric mapping [10], and are given by

$$a = (I + I^2)E_{\text{Rain}}(D^{w^2\phi^2}),$$

$$b = \frac{1}{2}(I + I^2)(E_{\text{Rain}}(D^{w^2\phi^2}) + E_{\text{Rain}}(D^{w^2\phi^2})), $$

$$c = E_{\text{Rain}}(D^{w^2\phi^2}) + IE_{\text{Rain}}(D^{w^2\phi^2}).$$

Thus, substituting (18) into (17) and performing the differentiation required in (16), gives the BER upper bound of 2-state TC-A8PSK system in the rain fading channel as

$$P_b \leq \left( \min_{\lambda \geq 0} \left( \min_{0 \leq \phi \leq 90^\circ} \right) \right) \frac{1}{n} \left\{ \left[ E_{\text{Rain}}(D^{w^2\phi^2}) \right] + \frac{\Delta}{1 - \left( E_{\text{Rain}}(D^{w^2\phi^2}) + E_{\text{Rain}}(D^{w^2\phi^2}) \right)} \right\},$$

where

$$\Delta = E_{\text{Rain}}(D^{w^2\phi^2}) + 3E_{\text{Rain}}(D^{w^2\phi^2}) \times \left( E_{\text{Rain}}(D^{w^2\phi^2}) + E_{\text{Rain}}(D^{w^2\phi^2}) \right) + 2E_{\text{Rain}}(D^{w^2\phi^2})E_{\text{Rain}}(D^{w^2\phi^2}) \times \left( 1 - \left( E_{\text{Rain}}(D^{w^2\phi^2}) + E_{\text{Rain}}(D^{w^2\phi^2}) \right) \right).$$
The first term and the second term of (19) account for the parallel transition and the non-parallel transition, respectively. Next, we consider the asymptotic BER behaviors of 2-state TC-A8PSK system. The usefulness of asymptotic analysis lies in the fact that it can tell us which parameter(s) of the code used dominate the system performance. Since the second term of (19) can be approximated to $E_{\text{Rain}}(D^{w^2\delta^2_1})$, thus the asymptotic result is given by

$$P_b \leq \left( \min_{\lambda \geq 0} \right) \left( \min_{0 \leq \phi \leq 90^\circ} \right) E_{\text{Rain}}(D^{w^2\delta^2_1}). \tag{21}$$

As previously mentioned, in 2-state TC-A8PSK system, we can obtain the additional gain due to asymmetric coded signal design. We will show it by using the numerical results in Sect.5 (see Fig.12).

4.2 4-State TC-A8PSK System

The 4-state TC-A8PSK encoder is presented in Fig.8(a). Unlike the 2-state TC-A8PSK system, since we cannot obtain the gain due to asymmetry in the 4-state TC-A8PSK system, we use the traditional symmetric coded signal sets [9]. The corresponding trellis diagram is presented in Fig.8(b). Since there also exists the parallel transition, we should consider the parallel transition to determine the transfer function of the error state diagram. The error state diagram of the 4-state TC-A8PSK system is illustrated in Fig.8(c). Using the Mason’s rule [10], analogous to (17), we obtain the following transfer function for the graph.

$$T(D, I) = IE_{\text{Rain}}(D^{w^2\delta^2_1}) + \frac{aeg(1 - c) + abdg}{1 - (c + ef + bdf) + cef}, \tag{22}$$

where the graph labels a–g of (22) are calculated by using the properties of isometric mapping in the same manner as in 2-state TC-A8PSK system, and are given by

$$a = (1 + I^2)E_{\text{Rain}}(D^{w^2\delta^2_1})$$

$$b = \frac{1}{2}(I + I^2) \left( E_{\text{Rain}}(D^{w^2\delta^2_1}) + E_{\text{Rain}}(D^{w^2\delta^2_2}) \right)$$

$$c = E_{\text{Rain}}(D^{w^2\delta^2_1}) + IE_{\text{Rain}}(D^{w^2\delta^2_2})$$

$$d = \frac{1}{2}(I + I^2) \left( E_{\text{Rain}}(D^{w^2\delta^2_1}) + E_{\text{Rain}}(D^{w^2\delta^2_2}) \right)$$

$$e = E_{\text{Rain}}(D^{w^2\delta^2_1}) + IE_{\text{Rain}}(D^{w^2\delta^2_2})$$

$$f = 1 + IE_{\text{Rain}}(D^{w^2\delta^2_1})$$

$$g = (I + I^2)E_{\text{Rain}}(D^{w^2\delta^2_2}). \tag{23}$$

Thus, substituting (23) into (22) and performing the differentiation required in (16), gives the BER upper bound of 4-state TC-A8PSK system in the rain fading channel as

$$P_b \leq \left( \min_{\lambda \geq 0} \right) \frac{1}{2} \left\{ E_{\text{Rain}}(D^{w^2\delta^2_1}) + E_{\text{Rain}}(D^{w^2\delta^2_2}) \right\}$$

$$+ \frac{E_1E_4 - E_2E_3}{E^2_1}, \tag{24}$$

where $E_1$, $E_2$, $E_3$, and $E_4$ are given by

$$E_1 = (1 - (c + ef + bdf) + cef)|_{I=1} = 1 - \left\{ E_{\text{Rain}}(D^{w^2\delta^2_1}) + E_{\text{Rain}}(D^{w^2\delta^2_2}) \right\}$$

$$\times \left\{ 2 + E_{\text{Rain}}(D^{w^2\delta^2_1}) \right\}$$

$$+ \left\{ 1 + E_{\text{Rain}}(D^{w^2\delta^2_2}) \right\}$$

$$\times \left\{ E_{\text{Rain}}(D^{w^2\delta^2_1}) + E_{\text{Rain}}(D^{w^2\delta^2_2}) \right\}^2$$

$$- \left\{ E_{\text{Rain}}(D^{w^2\delta^2_1}) + E_{\text{Rain}}(D^{w^2\delta^2_2}) \right\}^2. \tag{25a}$$
However, when the number of states in the trellis diagram becomes large, determining the error state diagram and its associated transfer function is a complex and boring task. Therefore, for the number of states in trellis diagram above eight, simulation is a more desirable method.

5. Numerical Results and Discussions

In this section, we present the numerical results of the BER upper bounds of TC-A8PSK system in the rain fading channel by using the analytic results shown in Sect. 4. To consider the effect of the rain fading, we use the mean $m_A$ and the standard deviation $s_A$ of $\log_{10} A$ which are calculated for the regions of the Crane global model. These parameters depend on both the bit energy to noise ratio ($E_b/N_o$) and elevation angle. Therefore, we analyze the effect of rain fading on the BER upper bounds of TC-A8PSK system in two different approaches. First, we can analyze the BER upper bounds as a function of the elevation angle $\varepsilon$ with $E_b/N_o$ fixed (see Figs. 9, 10, and 11). Second, we can analyze the BER upper bounds as a function of $E_b/N_o$ with the elevation angle fixed (see Figs. 12, 13, 14, and 15).

As a first approach, Fig. 9 shows the BER upper bounds of the 2- and 4-state TC-A8PSK system as a function of the elevation angle with $E_b/N_o$ of 15 dB. In this case, we assume that ideal channel state infor-
information (CSI) is available. The asymmetric angles of 2-state TC-A8PSK system should be optimized by minimizing (19), whereas the asymmetric angles of 4-state TC-A8PSK system are identical with the symmetric angle, that is, $\phi = 45^\circ$. Note that here the BER upper bounds of 2-state TC-A8PSK system give worse performance than those of the 4-state case. There exists a wider performance gap between the 2- and 4-state in the regions with a lower elevation angle and a larger rain attenuation. The Crane regions can be arranged in an order of higher rain attenuation as follows: $H$, $E$, $G$, $D$ ($= D_2$), $C$, $F$, $B$, and $A$. That is, the effect of the rain fading is least severe in the dry arctic region $A$, and is most severe in the wet tropical region $H$.

Next, to determine the optimum value of $\phi$ in the rain fading, we need to differentiate (19) with respect to $\phi$ and equate the result to zero. Rather than doing that, it is more expedient to directly minimize (19) with respect to $\phi$ by using numerical techniques. Figure 10 shows the variations of optimum asymmetric angles according to the elevation angle by considering Crane regional model. We assume that the ideal channel state information is available and the number of states in the trellis diagram is two. In this figure, the larger the elevation angle, the larger the optimum asymmetric angle, where optimum asymmetric angles have the range from $40^\circ$ to $80^\circ$. As a whole, the optimum asymmetric angles $\phi'$s for a dry regions (regions $A$, $B$, $C$, and $F$) producing less severe rain attenuation are larger than those for a wet regions (regions $D$, $E$, $G$, and $H$) producing more severe rain attenuation. As previously shown in Fig.9, we compared the BER upper bounds of 2- and 4-state TC-A8PSK system as a function of the elevation angle by assuming the ideal channel state information. In Fig.11, however, we illustrate the BER upper bounds of 4-state TC-A8PSK system as a function of the elevation angle according to the availability of channel state information. Here we assumed the bit energy to noise ratio $E_b/N_o$ of 15 dB and the asymmetric angle $\phi$ of $45^\circ$. Clearly, the lack of channel state information produces a noticeable degradation in system performance. And we also find the regional BER characteristics due to the rain attenuation.

As a second approach, we analyze the system performance as a function of $E_b/N_o$ with the elevation angle fixed. Figure 12 shows the BER upper bounds of 2-state TC-A8PSK system for region $D$ by assuming the ideal channel state information, where we assume the elevation angle of $45^\circ$. The system performances are severely impaired at the asymmetric angle below $45^\circ$. These results are in good agreement with that of the optimum asymmetric angles in the range of $40^\circ$ to $80^\circ$, as shown in Fig.10. The signal constellation of TC-A8PSK of asymmetric angle with $0^\circ$ and $45^\circ$ are equivalent to that for QPSK and symmetric 8PSK, respectively.

Figure 13 shows the BER upper bounds of the 2- and 4-state TC-A8PSK system as a function of $E_b/N_o$ with elevation angle $\varepsilon$ of $45^\circ$. In this case, we assume that ideal channel state information (CSI) is available. As indicated in Fig.9, there also exists a wider performance gap between the 2- and 4-state in the regions with a lower elevation angle and a larger rain attenuation. The regional BER characteristics with respect to $E_b/N_o$ are similar to those with respect to the ele-
Fig. 12 BER upper bounds of 2-state TC-A8PSK system versus asymmetric angle; ideal CSI.

Fig. 13 BER upper bounds of 2- and 4-state TC-A8PSK system versus $E_b/N_o$; ideal CSI.

Next, Fig. 14 shows the variations of optimum asymmetric angles of 2-state TC-A8PSK system versus $E_b/N_o$ by using numerical analysis. We assume that the ideal channel state information is available. Then, the larger the bit energy to noise ratio, the larger the optimum asymmetric angle. And optimum asymmetric angles have the range from 40° to 85°. Analogous to Fig. 10, the optimum asymmetric angles for a dry regions (region $A$, $B$, $C$, and $F$) are larger than those for a wet regions (region $D$, $E$, $G$, and $H$). Moreover, the smaller the Rician parameter $K$, the more severe the Rayleigh fading, and then, the larger the asymmetric angle. As a result, the distribution of the asymmetric angles in a less severe rain fading regions (regions $A$, $B$, $C$, and $F$) are similar to those of Rician fading environment with Rician parameter $K = 50$. Finally, in Fig. 15, we illustrate the BER upper bounds of the
4-state TC-A8PSK system as a function of $E_b/N_o$ according to the availability of channel state information. Here we assume that both the elevation angle and the asymmetric angle are $45^\circ$. Thus, we obtain the additional coding gain of about 0.4 dB due to ideal channel state information.

From the above numerical results, we summarize the dominant six factors which are closely related to the BER upper bounds of TC-A8PSK satellite system in the rain fading channel as follows. (1) frequency band: the high frequency bands including Ka-band are subject to severe performance degradations due to the atmospheric propagation fading, (2) rain intensity ($m_r$ and $s_r$): the rain intensity parameters are directly related to the rain attenuation parameters ($m_A$ and $s_A$) which determine the PDF of the rain fading, (3) elevation angle ($\varepsilon$): even if the rain intensities of the two earth stations are identical, the effect of the rain fading varies with elevation angle, (4) bit energy to noise ratio ($E_b/N_o$): the low $E_b/N_o$ directly causes the degradation of Viterbi decoder performance, (5) asymmetric angle ($\phi$): the additional coding gain due to asymmetry can be obtained by optimizing the asymmetric angle that minimizes the bit error rate, and (6) availability of CSI: the metric calculation of the Viterbi decoder depends on the availability of channel state information, which leads to a variation in system performance.

6. Conclusion

In this paper, we have presented the BER upper bounds of TC-A8PSK system using the Ka-band satellite in the rain fading channel. The PDF for the rain fading random variable could be theoretically derived by assuming that the rain attenuation can be approximated to a log-normal distribution. And the rain fading parameters were calculated by using the rain precipitation data from the Crane global model. Furthermore, we analyzed the BER upper bounds of TC-A8PSK system according to the number of states in the trellis diagram and the availability of CSI.

As a result of this investigation, we have found that the Crane regions can be arranged in an order of higher rain attenuation as follows: $H$, $E$, $G$, $D$ ($=D_2$), $C$, $F$, $B$, and $A$. That is, the effect of rain fading is least severe in the dry arctic region $A$, and is most severe in the wet tropical region $H$. The regional BER characteristics with respect to the bit energy to noise ratio are similar to those with respect to the elevation angle. As expected, the BER upper bounds of 2-state TC-A8PSK system give worse performance than those of the 4-state case. There exists a wider performance gap between the 2- and 4-state in the region with a lower elevation angle and a larger rain attenuation. Furthermore, we have found that optimum asymmetric angles have the range from 40° to 85° and the additional coding gain due to ideal CSI is about 0.4 dB. On the other hand, the degradation due to the rain fading is less severe than that due to the Rayleigh fading, but, is more severe than that due to the Rician fading with a Rician parameter $K = 10$ (typical case of the mobile satellite channel), specifically at the bit energy to noise ratio below 21 dB. Finally, we have summarized the dominant six factors which are closely related to the BER upper bounds of TC-A8PSK satellite system in the rain fading channel as follows: (1) frequency band, (2) rain intensity, (3) elevation angle, (4) bit energy to noise ratio, (5) asymmetric angle, and (6) availability of CSI.

These results can be utilized for the design of Ka-band satellite system by estimating the rain attenuation for the regions of interest. The accurate estimation of rain attenuation will enable system designers to plan a suitable amount of rain fading margin for a system, and will make it easier to satisfy the grade of service (or availability) requirements.

References

of the rain fading amplitude \( \rho \) as follows:

\[
\rho = \frac{1}{\sqrt{A}}, \quad \rho > 0. \tag{A·1}
\]

Solving (A·1) over the rain attenuation \( A \), we get

\[
A = \frac{1}{\rho^2}, \quad \rho > 0. \tag{A·2}
\]

The rain attenuation \( A \) can be approximated by log-normal distribution and have the form of \([6]\)

\[
f_A(A) = \begin{cases} 
\frac{C}{\sqrt{2\pi s_A^2}} e^{-\frac{(\log_{10} A - m_A)^2}{2s_A^2}} & ; \quad A > 0 \\
0 & \text{otherwise}
\end{cases} \tag{A·3}
\]

where \( C = \log_{10} e \approx 0.4343 \), \( e \) being the base of natural logarithm, and \( m_A \) and \( s_A \) are the average value and the standard deviation of \( \log_{10} A \), respectively.

Using the rain attenuation PDF \( f_A(A) \) of (A·3), the rain fading PDF \( f_\rho(\rho) \) can be given by \([11]\)

\[
f_\rho(\rho) = \left. \frac{f_A(A)}{\frac{dA}{\rho^2}} \right|_{A=1/\rho^2} = \frac{2f_A(1/\rho^2)}{\rho^3}, \quad \rho > 0. \tag{A·4}
\]

Finally, (A·4) becomes

\[
f_\rho(\rho) = \frac{2C}{\sqrt{2\pi s_A^2}} e^{-\frac{(\log_{10}1/\rho^2 - m_A)^2}{2s_A^2}}, \quad \rho > 0. \tag{A·5}
\]